

## ANNOTATED TRANSLATIONS

### MATHEMATICAL MATERIALS

- (26)  $M = \{x \mid x \in N\}$   
*M kye kibinja ky'ennamba x zonna zinnabutonde*  
 'M is the set of all natural numbers x'
- (27)  $x \in N$       *x nnakibinja mu N*  
 'x is a member of N'
- (28)  $A \subset B$       *A kibinja kito ddala ddala mu B*  
 'A is a proper subset of B'  
*A ddala ddala kizingirwa mu B*  
 'A is properly included in B'
- (29)  $A \cap B$       *ekisalaganiro kya A ne B*  
 'the intersection of A and B'
- (30)  $A \cup B$       *ekimuwazo kya A ne B*  
 'the union of A and B'
- (31)  $A \setminus B$       *A nga kitooleddwako B*  
 'A minus B'
- (32)  $A \setminus A = \emptyset$       *A nga kitooleddwako A kyenkana ekibinja ekyereere*  
 'A minus A equals the empty set'
- (33)  $\langle x, y \rangle$       *omugogo gwa x ne y omutegeke*  
 'the ordered pair of x and y'
- (34)  $A \times B$       *ekizaazise okuva mu A ne B*  
 'the set product of A and B'
- (35)  $P(A)$       *kinnabuyinza kya A*  
 'power set of A'
- (36a) 'f maps out of A into B'  
 (36b) 'f maps from A into B'  
 (36c) 'f maps out of A onto B'  
 (36d) 'f maps from A onto B'  
 (36e) 'f uniquely maps out of A into B'  
 (36f) 'f uniquely maps from A onto B'  
 (36a) *f kimaapuwaza okuva mu A nga kizza mu B*  
 (36b) *f kimaapuwaza okuva mu A nga kizza mu B*  
 (36c) *f kimaapuwaza okuva mu A nga kizza ku B*  
 (36d) *f kimaapuwaza okuva mu A nga kizza ku B*  
 (36e) *f kimaapuwaza bemmummululu okuva mu A nga kizza mu B*  
 (36f) *f kimaapuwaza bemmummululu okuva ku A nga kizza ku B*  
 or *f kimaapuwaza kimu-eri-kimu okuva ku A nga kizza ku B*
- (37) '*B is the complement of the set A*'  
*B kye kimalayo ky'ekibinja A*
- (38) 'U is the universal set'.  
*U kye kibinja kinnabyonnawazo*
- (39)  $f(x)$       'f of x'      *f gwa x*
- (40) 'f(x) is the image of x under the function f'

- $f(x)$  kye kifaananyi kya  $x$  wansi w'omukolo  $f$ .  
(41)    ' $f^{-1}$ ' is the inverse relation of the function  $f$   
       $f^{-1}$  kye kigandawazo nnagalika eky'omukolo  $f$
- Numbers are classified in (42)
- (42)     $C = R \cup I$   
       $Q = Z \cup F$   
       $N = P' \cup P$   
       $R = Q \cup Q'$   
       $Z = Z' \cup \{0\} \cup N$
- (43)     $C = \{a + bi \mid a, b \in R, i^2 = -1\}$   
      'complex number' ennamba zinnabuzibu
- (44)     $I = \{bi \mid b \in R, \{0\}, i^2 = -1\}$   
      'imaginary numbers' ennamba enfumiitirize
- (45)     $R$  'real numbers' ennamba wawu
- (46)     $Q = \{a/b \mid a, b \in Z, \{0\}\}$  'rational numbers'  
      ennamba emmenyefu kinnakkomo
- (47)    'irrational numbers' ennamba emmenyefu kinnabutakoma
- (48)     $Z = \{0, \pm 1, \pm 2, \dots\}$  'integers' ennambirira
- (49)     $F = \{a/b \mid a, b \in Z, \{0, 1\}\}$  'fractions' emmenyefu
- (50)     $Z' \{-1, -2, -3, \dots\}$  'negative integers' ennambirira egganya
- (51)    0 'zero' ezzeero
- (52)     $N = \{1, 2, 3, \dots\}$  'natural numbers' ennambirira zinnabutonde  
      'positive integers' ennambirira enzikirizi
- (53)     $P = \{2, 3, 5, 7, 11, 15, 17, 19, \dots\}$   
      'prime numbers' ennamba enkulu
- (54)    'nonprime numbers' ennamba zikkontankulu
- (55)    'The conjugate of  $a + bi$  is  $a - bi$ '  
      Ennywanyi ya  $a + bi$        $a - bi$
- (56)     $a + b$  'a plus  $b$ ' a ngatteko  $b$
- (57)     $a - b$  'a minus  $b$ ' a ntooleko  $b$
- (58)     $a \times b$  'a times  $b$ ' a emirundi  $b$
- (59)     $a + b$  'a divided by  $b$ ' a egabiddwamu  $b$
- (60)     $a > b$  'a greater than  $b$ ' a kusinga  $b$
- (61)     $a < b$  'a less than  $b$ ' a ntono ku  $b$
- (62)     $a \approx b$  'a approximately equal to  $b$ ' a kumpi kwenkana  $b$
- (63)     $a \gg b$  'a much greater than  $b$ ' a kusingira ddala  $b$
- (64)     $x \rightarrow \infty$  'x approaches infinity' x esemberera ( $\infty$ ) butakoma
- (65)     $4! = 24$  'Factorial 4 equals 24!' 4 kinnafakta 24
- (66)     $x^n$  'x (raised ) to the nth power.'  
      'x (nga eyimusibbwa okutuuka) ku buyinza bwa n
- (67)     $\sqrt[n]{x}$  'the nth root of  $x$ ' ekikolo kya  $x$  eky'a  $n$
- (68)     $y \propto x$  'y varies directly as  $x$ '  
      y ekyukakyuka butereevu nga  $x$
- (69)     $\lim f(x)$  'the limit of  $f(x)$  as  $x$  approaches  $a$ '

- $x \rightarrow a$  enkomerero ya  $f(x)$  nga  $x$  esemberera a  
(70) ( $A^{-1}$ ) 'the inverse of the non-singular matrix A'  
*enfuulannenge y'endoko A etali ya bwannamunigina*
- (71)  $A^T$  'the transpose of the matrix A  
*(endoko) enseetulule y'endoko A*
- (72)  $\det A$  'the determinant of the square matrix A  
*emmiimulula y'endoko A ey'omulabba*
- $\sum_{r=1}^n f(r)$  'the sum of  $f(r)$  to n terms'  
*omugatte gwa f(r) okutuuka ku bimiimo n*
- (74)  $\prod_{r=1}^n f(r)$  'the product of  $f(r)$  to n terms'  
*omuzaalise gwa f(r) okutuuka ku bimiimo n*
- (75)  $\log_a x$  'the logarithm of x to base a'  
*e<sup>YY</sup> andaganyo ya x ku musingi a*
- (76)  $\ln x$  'the natural logarithm of x  
*e<sup>YY</sup> andaganyo nnabutonde eya x*
- (77)  $dy/dx$  'the differential coefficient of y with respect to x'  
*omumuwendoganyo gwa y gunnamwawulo nga gufa ku x*
- (78)  $f'(x)$  'f prime of x'  
*f'kasale gwa x*
- (79)  $\int_a^b f(x)dx$  'the integral of  $f(x)$  with respect to x from a to b'  
*omulambirizo gwa f(x) nga gufa ku x okuva ku a okutuuka ku b*
- (80)  $P(A)$  'the probability of the event A'  
*obwandiba bw'ekituukiriro A*
- (81)  $P(A/B)$  'the probability of the event A conditional on the event B'  
*obwandiba bw'ekituukiriro A ku kakalu k'ekituukiriro B*

(Notes on 2.2.2 (26)-(81)]

"natural"; *obutonde*; "nature"; *-nnabutonde*  
 "member"; *ekibinja*; "set"; *-nnakibinja*  
 "intersection"; *-salaganir-*; "to intersect at" *ekisalaganiro*  
 "union"; *-mu*; "one"; *ekimuwazo*  
 "pair"; *omugogo*  
 "to map uniquely"; *-maapuwaz-* *bemmummululu*  
 "universal"; *byonna*; "all"; *-nnabyonnawaz-*  
 "inverse"; *-galik-*; "to invert"; *-nnaggalika*  
 "complex"; *obuzibu*; "difficulty"; *-nnabuzibu*  
 "imaginary"; *-fumiitirize*  
 "real"; *wawu*  
 "rational"; *-menyefu*; "broken finitely" *kinnakkomo*  
 "irrational"; *-menyefu*; "broken infinitely" *kinnabutakoma*  
 "fraction"; *-menyek-*; "to break"; *emmenyefu*  
 "prime"; *-kulu*  
 "nonprime"; *-kontankulu*

"determinant"; *-miimulul-*; "to tie very tightly"  
 "logarithm"; *-gabir-*; "to give to"; *olugabiro*  
 "coefficient"; *omuwendo*: "value"; *omumuwendoganyo*  
 "integral"; *-lambirira*; "whole"; *omulambirirawazo*  
 "probability"; *obwandiba*

Let the Greek alphabet from which symbols are widely adopted in logic, mathematics, physics and chemistry be assimilated to Luganda as in (82)

(82)	A	α	alpha	alfa
	B	β	beta	beta
	Γ	γ	gamma	gamma
	<	δ	delta	delta
	E	ε	epsilon	epsilononi
	Z	ζ	zeta	zeta
	H	η	eta	eta
	Θ	θ	theta	theta
	I	ι	iota	iota
	K	κ	kappa	kappa
	.	λ	lambda	lambuda
	M	μ	mu	myu
	N	ν	nu	nyu
	Ξ	ξ	xi	ksi
	O	ο	omicron	omikroni
	Π	π	pi	pi
	P	ρ	rho	ro
	Σ	σ	sigma	sigma
	Y	υ	upsilon	yupsiloni
	Φ	φ	phi	fi
	X	χ	chi	hyi
	Ψ	ψ	psi	psi
	Ω	ω	omega	omega

I render some mathematical texts into Luganda.

### (83a) Text 3: Differential Equations

#### Introduction

A differential equation is an equation which involves at least one derivative of an unknown function. The following are some examples of differential equations:

$$\begin{aligned} dy/dx &= \sin x \\ x \frac{dy}{dx} &= y^2 + 1 \\ \frac{dy}{dt} + y \frac{d^2}{dt^2} &= \cos t - e^y \\ x 2\frac{d^2y}{dx^2} - 2x \frac{dy}{dx} + 2y &= (x^2 - 2x + 2) e^x \end{aligned}$$

Many problems in physics, chemistry, engineering etc can be formulated in the form of differential equations. Thus differential equations play an important role in the application of mathematics to scientific problems.

#### Example 1

It is known that the rate of decay of a radioactive substance is proportional to the amount present. Express this in the form of a differential equation.

Solution

Let  $y$  be the amount of the radioactive substance present at time  $t$ . Then  $dy/dt$  is the rate of change. By assumption,  $d y/dt$  is proportional to  $y$ . Thus  $dy/dt = ky$  for some constant  $k$ .

Example 2

Newton's law of cooling states that the rate of change of temperature in a cooling body is proportional to the difference in temperature between the body and its surroundings.

Using  $t$  for time in minutes ,  $x$  for temperature of the cooling body in °C and  $x_0$  for the temperature of the surroundings (assumed to be constant), express the law in the form of a differential equation.

Solution

The rate of change of temperature is expressed as  $dx/dt$ . The difference in temperature between the body and its surroundings is given by  $x - x_0$ . Since  $dx/dt$  is proportional to  $x - x_0$ , we have  $dx/dt = k(x - x_0)$  for some constant  $k$ .

Example 3

Newton's law of gravitation states that the acceleration of a particle is inversely proportional to the square of the distance between the particle and the centre of the earth. Using  $x$  for that variable distance and  $t$  for time, express the law in the form of a differential equation relating to  $x$  and  $t$ .

Solution

Let  $v$  and  $a$  be respectively the velocity and acceleration of the particle at time  $t$ . Then

$$v = dx/dt, a = dv/dt$$

and hence

$$a = dv/dt = d/dt(dx/dt) = d^2x/dt^2$$

Since  $a$  is inversely proportional to  $x^2$ , we have

$$a = k/x^2$$

for some constant  $k$ . Thus, the required differential equation is

$$d^2x/dt^2 = k/x^2$$

[Notes on V.2.2(83)]

"engineering, technology"; *obukodyo*; "techniques" *kannabukodyo*

"science"; *okumanya*; "knowledge" *kannakumanya*

"decay"; *-seebengerer-*

"rate of decay"; *omuseebengerero*

"derivative"; *-viis-*; "to extract" *omuviiisemu*

"matter"; *omutole*; "lump" *nnamutole*

"radioactive"; *akagulu*; "ray" *-kaguluwazi*

"rate of change"; *-kyuk-*; "to change" *omukyuko*

'gravitation' ; *-sikiriz-*; "attract" *omusikirizo*

"gravitation"; *obuzito*; "weight" *nnabuzito*

"Earth"; *ettaka*; "earth" *Nnattaka*

"mathematics"; *okubala*; "(to) count(ing)" *kannakubala*

"distance"; *-esuul-*; "to be at a distance from obwesuulo

"to vary directly, to be proportional to"; *-kyuk-* *butereevu*

"to vary inversely, to be inversely proportional to"; -*kyuk-* *kinnaggalika*  
 "particle"; *akasirikitu*  
 "surroundings; -*etoolool-*; *obwetooloole*  
 "temperature" -*bugumy-*; "to be warmed" *obubugumye*

(83b) Translation of Text 3:

Ekyenkano kinnamwawulo kye kyenkano ekirimu waakiri omuviisemu ogumu ogw'omukolo ogutamanyiddwa. Ebiddirira bye bimu ku byokulabirako by 'ebyenkano binnamwawulo'

$$\begin{aligned} \frac{dx}{dt} &= \sin y \\ x \frac{dy}{dx} &= y^2 + 1 \\ \frac{dy}{dt} + y \frac{d^2y}{dt^2} &= \cos t - e^y \\ x 2\frac{d^2y}{dx^2} - 2x \frac{dy}{dx} + 2y &= (x^2 - 2x + 2) e^x \end{aligned}$$

Ebizibu bingi mu fizika, kemiya, kannabukodyo n'ebirala bisobola okwasanguzibwa mu kikula ky'ebyenkano binnamwawulo. Ebyenkano binnamwawulo bikulu mu nkozesya ya kannakubala ku bizibu bya kannakumanya.

Ekyokulabirako 1

Kimanyiddwa nti omuseebengerero gw'omutole ogukaguluwazi gukyuka butereevu nga obungi obuliwo. Yasanguza kino mu kikula ky'ekyenkano kinnamwawulo.

Ekimerengulo

Leka y bube obungi bw'omutole ogukaguluwazi oguliwo ku kiseera t. Olwo  $\frac{dy}{dx}$  gwe mukyuko. Kitwalibwa nti  $\frac{dy}{dx}$  gukyuka butereevu nga y. Nolwekyo  $\frac{dy}{dx} = ky$  kulwa ekitakyuka gundi k.

Ekyokulabirako 2

Etteeka ly'okuwola erya Newton ligamba nti omukyuko gw'obubugumye mu mubiri oguwola gukyuka butereevu nga enjawulo eriwo wakati w'obubugumye bw'omubiri n'obwetooloole. Nga okozesa t okuyimiririra ekiseera mu ddakiika, x okuyimiririra obubugumye bw'omubiri oguwola mu  ${}^{\circ}\text{C}$  ne  $x_0$  okuyimiririra obubugumye bw'obwetooloole (obutwalibwa nga tebukyuka), yasanguza etteeka mu kikula ky'ekyenkano kinnamwawulo.

Ekimerengulo

Omukyuko gw'obubugumye gwasanguzibwa nga  $\frac{dx}{dt}$ . Enjawulo mu bubugumye wakati w'omubiri n'obwetooloole bwagwo eweebwa na  $x - x_0$ , tufuna  $\frac{dx}{dt} = k_0 (x - x_0)$  olw'ekitakyuka gundi k.

Ekyokulabirako 3

Etteeka ly'a nnabuzito erya Newton ligamba nti omwanguyo gw'akasirikitu gukyuka kinnagalika nga omulabba gw'obwesulo wakati w'akasirikitu n'amassekati ga Nnattaka. Nga okozesa x okuyimiririra obwesulo obwo obukyuka ne t okuyimiririra ekiseera, yasanguza etteeka mu kikula ky'ekyenkano kinnamwawulo ekigandawaza x ne t.

Ekimerengulo

Leka v ne a mu buddiriggane obwo bibe embiro n'omwanguyo gw'akasirikitu ku kiseera t. Olwo  $v = \frac{dx}{dt}$ ,  $a = \frac{dv}{dt}$  era okuva awo  $a = \frac{dv}{dt} = \frac{d}{dt} \frac{dx}{dt} = \frac{d^2x}{dt^2}$  Okuva a lwe kikyuka kinnagalika nga  $x^2$ , tufuna  $a = k/x^2$  kulwa ekitakyuka k gundi. Nolwekyo, ekyenkano ky'omwawulo ekyetaagibwa kiri  $\frac{d^2x}{dt^2} = k/x^2$

(84a) Text 4

### Lagrangian equations

In the absence of constraints Lagrange's equations of motion (Eq..17.66) were found to be

$d/dt [\partial L/\partial q_i/\partial t] - \partial L/\partial q_i = 0$ , ne t (ekiseera) ekikyuka ekyetwala ne  $q_i(t)$  (akeesangiro k'akasirikitu) ekibinja ky'ebikyuka ebiteetwala. Ebitegekaganyo ttwalirawamu bitera okulondebwa amaanyi g'obukugire gasaanyizibwewo, naye kino tekiteekwa era ssi bulijo lwe kyetaagibwa. Nga ebikugiro  $\varphi_k$  we biri kinnamusingi wa Hamilton kiri with t (time) the one independent variable and  $q_i(t)$  (particle position) a set of dependent variables. Usually the generalized coordinates  $q_i$  are chosen to eliminate the forces of constraint, but this is not necessary and not always desirable. In the presence of constraints  $\varphi_k$  Hamilton's principle is

$$\int [L(q_i, dq_i/dt, t) + \sum \lambda_k(q_i, t) \varphi_k(q_i, t)] dt = 0, \quad (17.116)$$

and the constrained lagrangian equations of motion are

$$d/dt[\partial L/\partial q_i/\partial t] - \partial L/\partial q_i = \sum a_{ik}\lambda_k. \quad (17.117)$$

Usually  $\varphi_k = \varphi_k(q_i, t)$ , independent of the generalized velocities  $dq_i/dt$ . In this case the coefficient  $a_{ik}$  is given by

$$a_{ik} = \partial V/\partial q_i. \quad (17.118)$$

If  $q_i$  is a length, then  $a_{ik}\lambda_k$  (no summation) represents the force of the  $k$ th constraint in the  $q_i$ - direction, appearing in Eq.17.117 in exactly the same way as  $-\partial V/\partial q_i$ .

[Notes on V.2.2 (84)]

"constraint"; -*kugir-*; "to constrain" *ekikugiro*

"position"; -*esangir-*; "to find itself at" *akeesangiro*

"principle"; *omusingi*; "basis" *ekinnamusingi*

"coordinate"; -*tegekagany-* "to coordinate" *ekitegekaganyo*

(84b) Translation of Text 4:

### Ebyenkano by'Ekilagrange

Nga tewali bikugiro ebyenkano by'obwejjuluzi bya Lagrange (Kyenka. (17.66) byazuulibbwa okuba nga

$d/dt [\partial L/\partial q_i/\partial t] - \partial L/\partial q_i = 0$ , ne t (ekiseera) ekikyuka ekyetwala ne  $q_i(t)$  (akeesangiro k'akasirikitu) ekibinja ky'ebikyuka ebiteetwala. Ebitegekaganyo ttwalirawamu bitera okulondebwa amaanyi g'obukugire gasaanyizibwewo, naye kino tekiteekwa era ssi bulijo lwe kyetaagibwa. Nga ebikugiro  $\varphi_k$  we biri kinnamusingi wa Hamilton kiri

$$\int [L(q_i, dq_i/dt, t) + \sum \lambda_k(t) \varphi_k(q_i, t)] dt = 0, \quad (17.116)$$

era ebyenkano by'obwejjuluzi eby'Ekilagrange ebikugire biri

$$d/dt[\partial L/\partial q_i/\partial t] - \partial L/\partial q_i = \sum a_{ik}\lambda_k. \quad (17.117)$$

Kitera okuba nga  $\varphi_k = \varphi_k(q_i, t)$ , tebyesigama ku mbiro  $dq_i/dt$ . Mu ngeri eno omuwendowaganyo  $a_{ik}$  guweebwa na

$$a_{ik} = \partial \varphi_k/\partial q_i \quad (17.118)$$

Ssinga  $q_i$  buwanvu, olwo  $a_{ik}$  (tewali kugattirira) eyimiririra eryanyi ly'ekikugiro kya -k mu bwolekero bwa  $q_i$  erabika mu Kyenka. 17.117 mu ngeri ye nnyini nga  $-\partial V/\partial q_i$ .

